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APPLICATION OF A POINT ESTIMATE METHOD FOR INCORPORATING EPISTEMIC UNCERTAINTY IN THE SEISMIC ASSESSMENT OF A MASONRY BUILDING

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ABSTRACT

Several sources of uncertainty affect the assessment of existing buildings, including uncertainties associated with the material properties and the displacement capacity of the elements. In engineering practice, Monte Carlo simulations of the nonlinear seismic response of structures lead often to excessive computational costs and therefore rarely carried out. This paper proposes a simple logic tree approach, where a moment-matching technique is proposed to define the optimal sampling points and combination weights to apply to its branches. As a more refined method, a novel application to structural engineering of a recently proposed Point Estimate Method (PEM), which aims at reducing the required number of simulations further, is tested. Both methods are applied to a historical stone masonry building, which is modelled by an equivalent frame approach. The methods are benchmarked against the results of a Monte Carlo simulation and other approximate methods applied in the literature (FOSM, response surface method), which highlights the good accuracy of such methods for estimating the performance uncertainty of the tested building. Moreover, the effect of the different sources of uncertainty on the modelled performance of the building are discussed, identifying the displacement capacity as a major source of uncertainty, whose effect can be compared in terms of order of magnitude to the record-to-record variability.

Keywords: Point estimate method; epistemic uncertainty; stone masonry; seismic assessment

1. INTRODUCTION

Effective seismic risk management requires the development of assessment procedures that account, directly or indirectly, for the uncertainties that characterise the problem. The extent of these uncertainties is particularly relevant for existing buildings, and among those, for masonry historical constructions, due to the incomplete knowledge of the structures and materials that can be achieved, and to the peculiarities of their structural behaviour. Because this typology of buildings caused to a large part the casualties in recent seismic events (for instance, central Italy, 2016), a correct evaluation of their seismic performance is of great importance. However, in the professional practice, probabilistic assessment procedures including all relevant sources of uncertainty are seldom used, mainly for their complexity and computational cost.

A common approach in the seismic domain is the distinction between aleatory randomness and epistemic uncertainty (Fragiadakis and Vamvatsikos, 2010), attributed to the lack in knowledge and to the modelling approximations, often assumed uncoupled from the first. If the structural behaviour is correctly modelled and does not depend strongly on quantities affected by a large variability, the epistemic uncertainty is generally considered less relevant in comparison to the aleatory randomness (Lee and Mosalam, 2005). However, for masonry existing buildings, and for limit states close to collapse in particular, which are determined by the attainment of a limit displacement capacity of the elements (Dolsek, 2009; Vamvatsikos and Fragiadakis, 2010), these assumptions are not applicable and the

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epistemic uncertainty can have a considerable impact.
The treatment of epistemic uncertainty with a limited computational cost can be performed through simplified methods. The logic tree approach is an intuitive solution, that could be used in practice if the number of random variables is limited. In the literature (Bracchi et al., 2015; Franchin et al., 2010; Tondelli et al., 2012), it has been applied in particular to the treatment of sources of uncertainty that cannot be quantified by numerical random variables (modelling assumptions, for instance). The choice of branches and weights of a logic tree is a subjective choice; however, a consistent treatment of numerical variables, as presented in the following, can be derived.
Other methods have the scope of estimating directly the main properties of the response, namely the median response and its variance, through simplified approaches. A general strategy is approximating the response function with a surrogate model that can be determined through a limited number of evaluations. To this category belong First Order Second Moment methods (FOSM), making use of a Taylor series expansion of the response function around the mean, truncated to the second derivative (Pinto et al., 2004). Another approach is the Response Surface Method (RSM), consisting in the approximation of the response function by a first or second order polynomial, proposed in its linear formulation also by a document issued by the Italian CNR (2013).
As an alternative to these approaches, still requiring a limited number of evaluations of the model, Point Estimate Methods (PEM), or, equivalently, moment matching techniques, are a viable and consistent solution for the evaluation of the uncertainty in the seismic assessment of a building. In this paper, a novel PEM proposed by Franceschini et al. (Franceschini et al., 2012) is tested through a case study, in a first application to structural engineering, as a possibility of including epistemic uncertainty through simple procedures applicable also in engineering practice.
The results are compared to methods of similar computational cost (FOSM, RSM), and benchmarked against a Monte Carlo simulation. The influence of different sources of uncertainty is investigated, including the effect of spatial variability of properties among the elements of the structure, and its consideration through a simplified method.

2. POINT ESTIMATE METHODS

2.1 Selection of optimal estimation points

The central idea of Point Estimate Methods is the application of a suitable integration scheme to approximate some of the integrals that define the central moments of a generic function of a set of random variables, typically limiting the field of interest to the expected value and the variance. In the applied case, the input variables are the parameters defining the epistemic uncertainty, treated as random variables, and their output function is the seismic capacity of the corresponding structure, evaluated in terms of Peak Ground Acceleration (PGA).
The need of limiting the number of required analyses, keeping an acceptable level of accuracy, imposes the choice of effective integration schemes. These schemes correspond to the discretization of the continuous random variables into a limited set of evaluation points, to which optimal integration weights are assigned. Points and weights are determined imposing the correspondence between the first statistical moments of the original continuous distribution and the discrete one; for this reason these techniques can be referred as moment-matching methods.
The underlying assumption is that the central moments of a generic function of a random variable with a certain distribution are best approximated by the same function of a different variable, whose distribution matches as many central moments as the original. This can be formalized, in the case of a scalar function g of a single random variable X, by writing the Taylor-series expansion of the expected value of g around the mean of X, μ_X, and using the linearity of the operator $E[\cdot]$:

$$E[g(X)] = g(\mu_X) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n g}{\partial x^n}\bigg|_{\mu_X} E[(X-\mu_X)^n] = g(\mu_X) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n g}{\partial x^n}\bigg|_{\mu_X} \mu_X^{(n)} \quad (1)$$

where $\mu_X^{(n)}$ is the central moment of order n of X. The expression shows that by substituting the variable X with another random variable $\tilde{X}$ matching the first k central moments of the original distribution, one obtains the same series expansion, truncated to the *k*-th term. For numerical applications, clearly, $\tilde{X}$ would be a discrete distribution defined by a set of few points and their relative weights. Therefore, if the function *g* is approximated exactly by a Taylor series expansion of order *k*, its expected value can be estimated exactly substituting the original random variable with $\tilde{X}$.

Equivalent expressions of the one shown in Equation (1) can be derived for higher order statistical moments, with the difference that a higher number of moments would need to be matched for an exact estimation. For example, assuming that *g* is approximated exactly by a series expansion with *k* terms, the variance is evaluated exactly if 2*k* central moments are matched. The same procedure can be followed to derive expressions for the statistical moments of a multivariate series expansion, in the case of functions of several uncorrelated random variables (Ching et al., 2009).

This approach, that can be termed as a Gaussian quadrature scheme (Christian and Baecher, 1999), requires the identification of the points x_i and weights p_i to assign to the discrete distribution to match the maximum number of central moments. To formulate the problem it is convenient to write x_i as a function of the mean and standard deviation of the original distribution (respectively μ_x and σ_x), as in Equation (2). If the discrete distribution consists of *m* points, one can write 2*m* conditions to impose the correspondence of the first 2*m*-1 central moments, as expressed in the system of Equations (3).

$$x_i = \mu_x + \xi_i \sigma_x \tag{2}$$

$$\begin{cases} p_1 + p_2 + \dots + p_k = 1 \\ p_1\xi_1 + p_2\xi_2 + \dots + p_k\xi_k = 0 \\ p_1{\xi_1}^2 + p_2{\xi_2}^2 + \dots + p_k{\xi_k}^2 = \dfrac{\mu_x^{(2)}}{\sigma_x^2} = 1 \\ \vdots \\ p_1{\xi_1}^{2m-1} + p_2{\xi_2}^{2m-1} + \dots + p_k{\xi_k}^{2m-1} = \dfrac{\mu_x^{(2m-1)}}{\sigma_x^{2m-1}} \end{cases} \tag{3}$$

A numerical solution of this kind of problem was proposed by Miller and Rice (1983). The optimal sample points, defined by the coefficients ξ_i, are the roots of the polynomial defined in Equation (4), where the set of *m* unknown constants C_i is found solving the linear system in Equation (5)

$$\pi(\xi) = C_0 + C_1\xi + C_2\xi^2 + \dots + C_{m-1}\xi^{m-1} + \xi^m = 0 \tag{4}$$

$$\begin{bmatrix} 1 & 0 & \dfrac{\mu_x^{(2)}}{\sigma_x^2} & \dots & \dfrac{\mu_x^{(m-1)}}{\sigma_x^{m-1}} \\ 0 & \dfrac{\mu_x^{(2)}}{\sigma_x^2} & \dfrac{\mu_x^{(3)}}{\sigma_x^3} & \dots & \dfrac{\mu_x^{(m)}}{\sigma_x^m} \\ \vdots & & \ddots & & \\ \dfrac{\mu_x^{(m-1)}}{\sigma_x^{m-1}} & \dfrac{\mu_x^{(m)}}{\sigma_x^m} & & & \dfrac{\mu_x^{(2m-2)}}{\sigma_x^{2m-2}} \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{m-1} \end{bmatrix} = - \begin{bmatrix} \dfrac{\mu_x^{(m)}}{\sigma_x^m} \\ \dfrac{\mu_x^{(m+1)}}{\sigma_x^{m+1}} \\ \vdots \\ \dfrac{\mu_x^{(2m-1)}}{\sigma_x^{2m-1}} \end{bmatrix} \tag{5}$$

However, the position of the sampling points, optimally determined by Equations (4)-(5), could be imposed a-priori (Ching et al., 2009), or partially corrected by the choice of the analyst (for instance, if some points are sampled outside the region of interest). In all cases, the set of weights to be assigned to the sample points can be directly derived as the solution of the first *m* equations of the system in Equation (3). The linear system to be solved is given by:

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ \xi_1 & \xi_2 & & \xi_m \\ \vdots & & \ddots & \\ {\xi_1}^{m-1} & {\xi_2}^{m-1} & & {\xi_m}^{m-1} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \dfrac{\mu_x^{(m-1)}}{\sigma_x^{m-1}} \end{bmatrix} \tag{6}$$

As discussed, the solution of the problem cannot be derived in explicit form for a generic distribution and an unknown number of evaluation points, but several simplifications apply if some assumptions on the distribution of the random variable are made (normality, for instance). However, for the cases that are more relevant for applications in the multivariate case, i.e. a two- or three-point discretization, closed-form solutions can be derived. When only two points are sampled, the first three central moments can be matched, and the expressions in Equation (7) are obtained as a function of the skewness λ. For symmetrical distributions (λ=0), these expressions give the standard sampling at $\pm 1\sigma$, with equal weights, that is commonly applied in factorial design schemes.

$$\begin{cases} \xi_1 = \lambda/2 - \sqrt{\left(\lambda/2\right)^2 + 1} \\ \xi_2 = \lambda/2 + \sqrt{\left(\lambda/2\right)^2 + 1} \end{cases} \qquad \begin{cases} p_1 = \xi_2/(\xi_2 - \xi_1) \\ p_2 = 1 - p_1 \end{cases} \tag{7}$$

The three-point discretization, which will be applied in this study, has no closed-form general solution, unless the position of the centre point is fixed, conveniently, to the mean value. This assumption indeed will allow an easier combination of the variables when the full factorial combination is not applied, although reducing the number of central moments that can be matched to four instead of five. The position and weights to assign are given in Equation (8), where κ is the kurtosis. The optimal sampling of a normal distribution is obtained for points at $\xi_{1,3}=\pm 1.732$ and weights equal to $p_1=p_3=1/6$.

$$\begin{cases} \xi_1 = \lambda/2 - \sqrt{\kappa - 3/4\,\lambda^2} \\ \xi_2 = 0 \\ \xi_3 = \lambda/2 + \sqrt{\kappa - 3/4\,\lambda^2} \end{cases} \qquad \begin{cases} p_1 = -1/\xi_1 \cdot (\xi_3 - \xi_1) \\ p_2 = 1 - p_1 - p_3 \\ p_3 = 1/\xi_3 \cdot (\xi_3 - \xi_1) \end{cases} \tag{8}$$

2.2 Reduced integration schemes

The consistent application of Gaussian quadrature schemes in the multivariate case would require the use of a full factorial combination scheme. However, if the number of random variables that control the problem is N and the number of sampling points is m, the number of evaluations needed of the output function can rapidly grow as m^N. If each evaluation corresponds to a costly nonlinear analysis, as would be the case in seismic design, the number of analyses can easily be too large for practical applications if more than 4-5 different sources of uncertainty are evaluated together.

To overcome this drawback, when the application of a full factorial combination is excessively demanding, reduced combination schemes are available and constitute a viable alternative, which introduces a minimum approximation. Following the work of Rosenblueth (1975, 1981), several other PEMs were proposed (Hong, 1998; Li, 1992; Zhao and Ono, 2000), defining expressions to estimate the first central moments of an output function with the use of up to $2N$ or $2N$+1 evaluations of the response. A common assumption, corresponding to star schemes in factorial design, consists in assuming that only one random variable changes at a time. This, however, although reducing considerably the numerical cost, limits at the same time the insight of the combined effects of the random variables.

In this context the formulation proposed more recently by Franceschini et al. (2012) will be applied and compared to the complete combination. This formulation, requiring $2N$+1 evaluations and a sampling of the random variables on 3 levels, allows solving some of the problems of other PEMs, such as the theoretical possibility of estimating a negative variance or the excessive simplification that a two-point estimate of the input distributions can imply. The expressions for the mean and the variance of the output function g are given in Equations (9)-(11). The superscripts $i+$ and i- correspond, respectively, to the positive and negative variations of the i-th random variable, when all the others are kept equal to their mean value.

$$\mu_g = \left[1 - \sum_{i=1}^{N} (p_{i-} + p_{i+})\right] \cdot g^0 + \sum_{i=1}^{N} \left[p_{i-} g^{i-} + p_{i+} g^{i+}\right] \tag{9}$$

$$\sigma_g^2 = \sum_{i=1}^{N} \left[p_{i-} (g^{i-} - g^0)^2 + p_{i+} (g^{i+} - g^0)^2 - (p_{i-} + p_{i+}) e_i^2\right] \tag{10}$$

$$e_i = p_{i-} g^{i-} + p_{i+} g^{i+} - (p_{i-} + p_{i+})\, g^0 \tag{11}$$

3. APPLICATION TO THE SEISMIC ASSESSMENT A STONE MASONRY BUILDING

The performance of both the presented integration schemes, i.e. the complete Gaussian quadrature and the reduced integration as formulated in Equations (9)-(11), was compared to alternative methods (FOSM, RSM), of similar numerical cost, for the evaluation of the epistemic uncertainty in a practical application. Since the method is proposed to be a feasible alternative for applications also out of the research field, the complexity of the analyses was limited to nonlinear static analyses.

3.1 Building model

The building chosen as a case study is the Holsteiner Hof, a stone masonry building in city centre of Basel, Switzerland, which was built in the 17th century as a noble residence. Its structural simplicity and the regularity of the layout make it a suitable example for building a robust structural model, that could be applied in multiple nonlinear analyses, as required by a Monte Carlo simulation, and ease the automatic processing and the interpretation of the results. Nonetheless, the proposed method can be applied effectively also to more complex layouts, in which a higher degree of nonlinearity of the response function can be expected, since it relaxes, as presented, some assumptions that are at the base of the formulation of other methods, as the linear RSM.

The building is a two-storey stone masonry construction, schematically shown in Figure 1a, with plan dimensions of 14x26 m, assumed for the scope of this analysis as free on all four sides, although in its real configuration some minor buildings, built successively, are present at one of its shorter sides. Openings are laid in a regular grid that allows defining a frame of piers and spandrels. A gable is present in both longer façades.

The thickness of the walls decreases at the upper floors, ranging from 60 cm at the base to 30 cm of the last floor. The thickness of the walls under the windows is reduced to 15 cm, and this is accounted for when defining an equivalent thickness for the spandrels, modelled with single elements. Thinner partition walls, which are timber frame walls with brick infills, are present in the interior but are not modelled in the present analyses, as they are unlikely to influence the seismic response of the building in a significant manner. It is clear that the distinction between structural and non-structural elements is somehow subjective, and should in general be avoided for real applications to historical buildings. The objective of this study is, however, the comparison of different methods more than the assessment of this specific building in its actual configuration.

Horizontal diaphragms are made of deformable timber floors, with principal beams laid in the direction of the short sides, and a single layer of nailed planks. Additional 20 cm of non-structural material are assumed to be present. Floors are disposed at a 4.50 m inter-storey height. The roof is a complex truss structure, not explicitly modelled, assumed to apply only vertical forces to the underlying walls.

The adopted modelling strategy is an equivalent frame modelling of the building (Figure 1b), where the in-plane nonlinear behaviour of piers and spandrels is simulated through the macroelement formulation proposed by Penna et al. (2014), and the areas of masonry between piers and spandrels are treated as rigid nodes. All simulations were run using the software Tremuri (Lagomarsino et al., 2013). Diaphragms are modelled though elastic orthotropic membrane elements, providing a different in-plane stiffness in the two directions of the timber floor; a sufficiently good connection with the walls is simulated, with no possible sliding or loss of support of the beams.

Global pushover analyses are carried out, where the Ultimate Limit State (ULS) is considered attained when the first pier reaches a drift corresponding to a near-collapse condition. It should be noted that in a real case scenario the assessment of local out-of-plane mechanisms would be definitely relevant, since the stiffness of the diaphragms and their connection to the walls cannot be assumed to be sufficient to prevent their activation. However, the epistemic uncertainty in the assessment of such mechanisms is related mainly to the modelling assumptions, rather than to quantities that can be modelled as random variables, and for this reason, given the scope of the present work, their analysis is here omitted. The adoption of a software available to a part of the professional community, and the use of simple pushover analyses, aims at developing a methodology that could be applied in the near future also by practitioners.

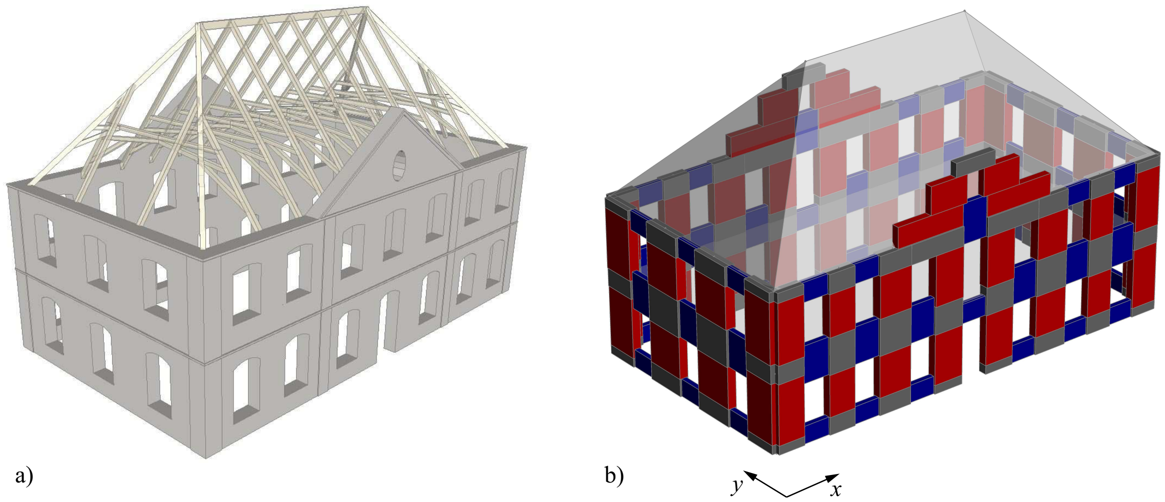

Figure 1 (a) Structural scheme of the Holsteiner Hof building in Basel, Switzerland; (b) macroelement equivalent frame model (Tremuri) used for the simulations

3.2 Sources of uncertainty

The main steps when assessing the epistemic uncertainty in an analysis are the identification of the main sources of uncertainty to be modelled, and their quantification through the distribution assigned to a random variable. The first step can be dealt with through a sensitivity analysis, which also helps in defining an plan for in-situ tests in order to reduce the total epistemic uncertainty (Cattari et al., 2015). If no specific in-situ test results are available, the quantification of the distributions, has to be based on literature or code provisions. The sources of uncertainty individuated for the present study are listed in Table 1, together with the source from which their distribution was assumed.

For the mechanical properties of stone masonry the table provided by the Italian code (MIT, 2009) is considered as a reference, according to the indications provided also by Kržan et al. (2014). The ranges given in the code, consistently with the provisions in CNR (2013), are interpreted as 16^{th} and 84^{th} fractiles of a lognormal distribution. These references differentiate between five different typologies of stone masonry; as an additional source of epistemic uncertainty, the correct attribution of the masonry to one of the classes defined in the code is here considered, because if the knowledge of the masonry typology is incorrect or partial, a misclassification can occur. The adopted distributions of mechanical properties correspond to the hypothesis that the quality of the real masonry can range from rubble masonry to cut-stone irregular masonry with good bond.

The definition of the mechanical behaviour of the macroelement used in the analysis requires also the definition of the friction coefficient, since its shear capacity corresponds to a Mohr-Coulomb criterion. The distribution used for this parameter, uniform between 0.2 and 0.3, is adopted from Angelillo et al. (2014); in Vanin et al. (2017) comparable values are derived. When the friction coefficient is calibrated against cyclic experimental tests, the ratio between cohesive and frictional contributions defines the hysteretic behaviour. In the case in which the cyclic response is of interest, more conservative values were assumed in the literature, which were as low as 0.08-0.14 (Penna et al., 2015; Rota et al., 2010).

Since the macroelement formulation adopted here accounts explicitly for a stiffness decrease due to decompression or shear damage, an effective stiffness is not required and the values in Table 1 are directly applied.

Table 1 Distributions adopted for the random variables

Parameter		Mean	COV.	Distribution	Reference
Elastic modulus	E	1280 MPa	0.35	Lognormal (μ=7.10, σ=0.34)	(CNR, 2013; Kržan et al., 2014)
Shear modulus	G	430 MPa	0.35	Lognormal (μ=6.10, σ=0.34)	(CNR, 2013; Kržan et al., 2014)
Compressive strength	f_c	2.39 MPa	0.45	Lognormal (μ=0.78, σ=0.43)	(CNR, 2013; Kržan et al., 2014)
Shear strength	τ_0	0.045 MPa	0.45	Lognormal (μ=-3.19, σ=0.43)	(CNR, 2013; Kržan et al., 2014)
Friction coefficient	μ	0.25	0.12	Uniform (a=0.2, b=0.3)	(Angelillo et al., 2014)
Failure drift, flexure	δ_{fl}	1.47%	0.57	Lognormal (μ=0.24, σ=0.53)	(Vanin et al., 2017)
Failure drift, shear	δ_{sh}	1.12%	0.69	Lognormal (μ=0.08, σ=0.62)	(Vanin et al., 2017)
Floor stiffness ratio	E_2/E_1	50%	0.55	Uniform (a=2.5%, b=100%)	

The displacement capacity of masonry structural elements is known to be a fundamental parameter controlling the seismic behaviour of a building. Codes currently do not provide indications of the uncertainty related to the estimation of the drift capacity, nor takes into account the typology of masonry in its definition. However, a considerable range of dispersion of the displacement capacity of masonry piers is reported in the literature (Kržan et al., 2014; Vanin et al., 2017), making the uncertainty related to this quantity the most relevant when a displacement-based assessment of a masonry building is performed. In the present study, the distribution of the drift capacity at near-collapse limit state are derived from the database of cyclic shear-compression tests on stone masonry walls provided in Vanin et al. (2017), using only tests on the masonry typologies here considered. Given the large dispersion of the drift capacities, a lognormal distribution has to be adopted.

The stiffness of the timber floors was considered as an additional source of uncertainty. The analyses are performed in the weakest direction of the floors, perpendicular to the orientation of the principal beams. Simple mechanical models are available for estimating the stiffness of timber diaphragms (Brignola et al., 2008), based on the stiffness of the timber elements and the connection elements. Since no continuous timber elements connect the two sides of a floor in the direction perpendicular to the beams, the stiffness in this direction is particularly controlled by the stiffness of the connections (nails in this case), that is in turn affected by a large uncertainty. In addition to this, the properties of the floor-to-wall connection are not simply determined. For these reasons, the stiffness adopted for the elastic membrane in the numerical analysis is assumed as a ratio of the stiffness in the direction of the beams, uniformly ranging from 0.025 to 1. The lower bound is set to avoid modelling a local overturning mechanism, which would develop if a zero stiffness was imposed, which could not be reliably simulated by the adopted numerical model.

In one Monte Carlo simulation, a spatial variability of the quantities defined by the random variables is assumed, while in the other analyses the same properties are assigned to all elements. When the spatial variability is accounted for, the properties attributed to the single elements are sampled from a distribution that is symmetric to the average value and has a COV of 20%. The value is estimated from a limited number of observations of intra-series variability in repeated tests (Vanin et al., 2017).

4. EVALUATION OF THE EPISTEMIC UNCERTAINTY WITH DIFFERENT METHODS

The simulations that were tested for the evaluation of the epistemic uncertainty on the seismic performance of the Holsteiner Hof building, under the presented hypotheses, were:

- two benchmark Monte Carlo simulations, respectively with and without spatial correlation of the random variables defining the response of the elements (550 analyses);
- Latin Hypercube Sampling using a small number of simulations (18 analyses)
- partial factorial combination (“logic tree” approach, 18 analyses)
- PEM, RSM, FOSM (7 analyses each)

Latin Hypercube Sampling, as a technique often applied to optimise the random sampling of multiple variables (Dolsek, 2009), was implemented through the Simulated Annealing algorithm as defined in Vorechovsky and Novak (2003). It was applied both to the complete Monte Carlo simulations and to a considerably reduced Monte Carlo analysis of the same numerical cost as the reduced schemes that are tested. Although it is meant to be used in much larger simulations, its performance, compared to the other techniques, is here evaluated.

Reduced schemes require the adoption of a limited number of random variables, if the total number of analyses has to be kept reasonably low. In order to simplify the problem, three major sources of uncertainty were defined, namely the uncertainty related to the mechanical parameters, to the displacement capacity, and to the diaphragm properties. In order to do so, perfect correlation has to be assumed between all variables belonging to the same category (stiffness, compressive strength and shear strength for the first, flexural and shear failure drift for the second).

These three random macro-variables were sampled according to their distribution in three points each, optimally determined by Equation (8). Since the influence of the floor stiffness is relevant only for low stiffness ratio values, a two-point sampling was applied, fixing the position of one of them and solving the problem of Equation (3). A consistent application of the Gaussian quadrature scheme, as discussed, would require a full factorial combination of the three variables on the sampled points (requiring totally 18 analyses). This approach can be visualised, for ease of application, as a logic tree, in which all combinations are considered and the weight assigned to each analysis is the product of the weights of the branches to which it belongs.

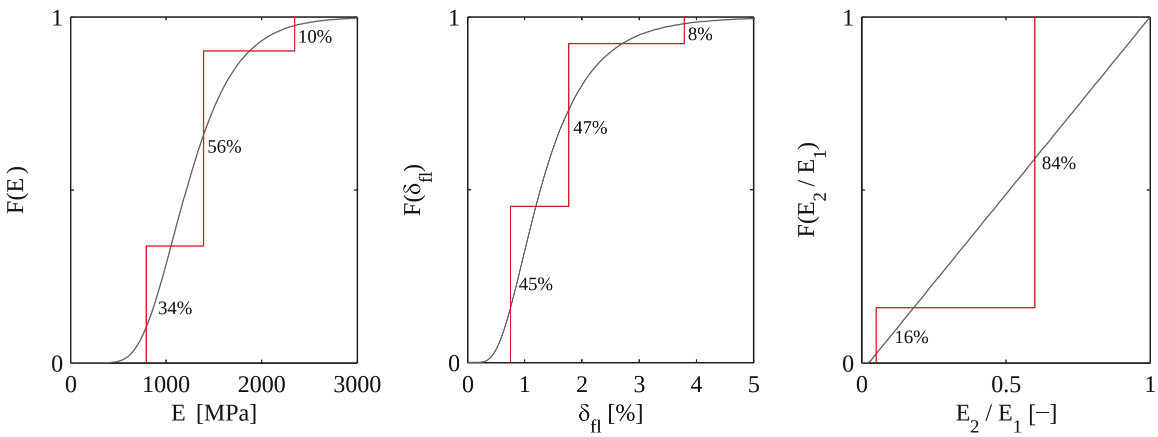

Figure 2 Sampling of the three random macro-variables defining the mechanical properties (left), the displacement capacity (centre) and the floor stiffness (right)

The comparison between this logic tree approach and the reduced Monte Carlo simulation with 12 analyses determined by LHS is graphically shown in Figure 3. The smooth approximation of the true cumulative distribution is obtained calculating the lognormal distribution that has the same mean value and variance of the discrete distributions of PGA. The reference Monte Carlo simulation does not include any spatial variability of the parameters. Both approaches lead to a satisfying estimate of the variability due to epistemic uncertainty, with a slightly better performance of the logic tree approach, in particular in the upper tail of the distribution.

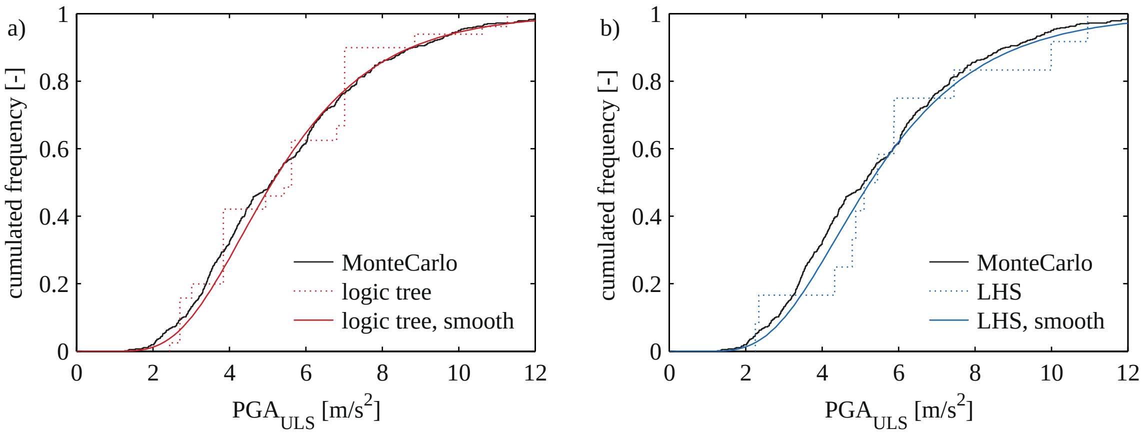

Figure 3 Cumulative distributions of the PGA that corresponds to ULS estimated with a logic tree approach (left) and a reduced Monte Carlo analysis through LHS (left)

As shown in Figure 4 the consideration of spatial variability can lead to a considerable shift of the median response, particularly if a criterion relating the failure of the first element to the global attainment of the limit state is applied, as it was done here. However, even if other failure criteria are considered, for instance a 20% drop of lateral capacity, the effect of spatial variability would be significant as well, since the response is determined by a floor mechanism, rather than by a local failure mode.
The most relevant parameter in determining the shift of the distribution of the response is the displacement capacity of the elements, as shown by the limited difference between the analyses in which spatial variability is considered only for the drift capacity (black dotted line) or for all random macro-variables (full black line). Since the failure criterion corresponds to the attainment of a displacement capacity limit, the model can be approximated as an in-series system, where the only property that varies between the elements is their drift capacity.
Following this approach, a model that includes spatial variability of random variables can be approximated by models in which all elements are assigned the same properties, if their values are adjusted. This corrected value can be computed, from the distribution describing the spatial variability of a certain property, as the value corresponding to the fractile f in Equation (12), where N_e is the number of elements. The application of this criterion for both the logic tree approach and the reduced Monte Carlo simulation is shown in Figure 4.

$$f = 1 - 2^{-\frac{1}{N_e}} \qquad (12)$$

Less demanding methods for the estimate of the epistemic uncertainty are PEM, RSM and FOSM. Such methods require a number of simulations determined by a star scheme, where each random macro-variable varies between an upper and lower value, and all others remain equal to their mean. An additional point corresponding to the mean of all variables has to be added, for a total of $2N$+1 simulations. For the RSM and FOSM methods a standard sampling of each variable at its 16th and 84th percentile is applied, which is consistent with the guidelines in CNR (2013), while for the PEM analysis the sampling and the corresponding weights are determined by Equation (8).
The comparison between the mean value and standard deviation of the response, predicted with the different methods, is presented in Table 2. The approach that leads to the best estimate is the logic tree, corresponding to a complete Gaussian quadrature scheme. Methods requiring a lower number of analyses show, as expected, a smaller level of accuracy, with the estimate of the variance being affected by the larger errors. The best estimate of the variance of the response, among the methods requiring $2N$+1 analyses, is the PEM for this case study.
A final remark regards the quantification of the epistemic uncertainty for the analysed case. As discussed, the response of the building, for the adopted hypotheses, is controlled by the displacement capacity of its elements. As a result of the relatively large uncertainty by which the drift capacity of

stone masonry piers is affected (Vanin et al., 2017), the uncertainty characterising the seismic response of the building is considerable. A quantification of this uncertainty in terms of lognormal variance gives a value of β=0.42 for this case study, which is comparable to the uncertainty related to the record-to-record variability, which can assumed in the order of 0.30-0.40 (Vamvatsikos and Fragiadakis, 2010).
It should be pointed out that this study considers only in-plane failure modes. The assessment of local out-of-plane mechanisms is in turn affected by a considerable epistemic uncertainty, related mainly, however, to the modelling assumptions rather than parameter estimation. Therefore, the quantification of this type of uncertainty is less objective, and out of the scope of the present work.

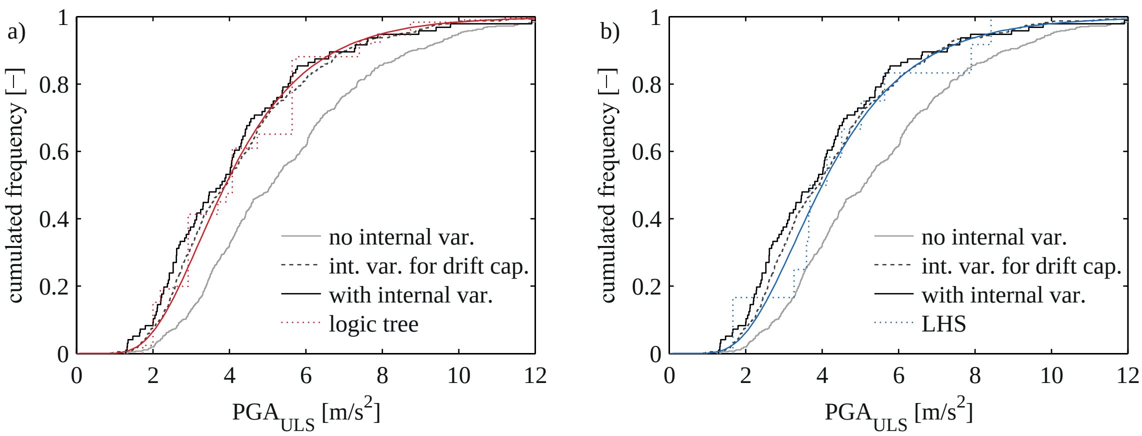

Figure 4 Effect of the spatial variability of random variables approximated through a logic tree approach and a reduced Monte Carlo simulation

Table 2 Comparison between the modelled mean value and standard deviation of the response.

	Analyses	Mean [m/s²]		Standard dev. [m/s²]	
Reference Monte Carlo	550	5.48		2.43	
Logic tree approach	18	5.46	-0.3%	2.38	-1.8%
Reduced Monte Carlo (LHS)	18	5.44	-0.8%	2.57	+5.8%
Point Estimate Method	7	5.41	-1.3%	2.39	-1.7%
Response Surface Method	7	5.82	+6.3%	2.81	+15.6%
FOSM	7	5.39	-1.6%	2.98	+22.9%

5. CONCLUSIONS

Different methods for the evaluation of the epistemic uncertainty in the seismic assessment of a stone masonry building were tested and compared, including a novel application of a Point Estimate Method (PEM) to the domain of seismic engineering as wells as a more intuitive, although more demanding, logic tree approach. The application of such methods lead to comparable or more accurate estimates of the uncertainty (particularly of the variance of the response), when compared to other simple methods, such as RSM and FOSM, and were benchmarked against a more extensive Monte Carlo simulation
The approximation of a random variable with a discrete set of points and relative weights was discussed, adopting as a criterion the match of as many statistical moments as possible between the original and the approximated distribution. The resulting optimal sampling points and combination weights are derived formally and checked in an application to a real case study. Different criteria largely adopted in the literature, such as the standard choice of a two-point discretisation at one standard deviation distance from the mean, impose less freedom to the analyst and are optimal only for normal distributions. However, since many material and structural parameters are affected by a large uncertainty and must be non-negative, the adoption of lognormal distributions is rather common, and therefore optimal

discretisation of such distributions can be effectively applied.
A Gaussian quadrature scheme, through a complete factorial combination, which corresponds to the logic tree approach, provides the best estimate of the uncertainty. The superiority of such technique, in comparison to methods requiring the linearization of the response function (RSM) or the estimation of the first two derivatives (FOSM), can be proven formally showing the possibility of approximating higher order terms in a Taylor series expansion of the response function. However, due also to the regularity of the case study and the reduced nonlinearity of the response, also less refined methods provided satisfactory uncertainty estimates. Among the methods requiring a lower number of analyses, the PEM, for the consistent derivation of the variance estimate from a Taylor series approximation, is the method that best captured the distribution of the response function.
The epistemic uncertainty in the seismic assessment of the studied building is shown to be strongly dependent on the in-plane displacement capacity of the stone masonry elements. The uncertainty related to the determination of this quantity results in a quantification of the global epistemic uncertainty that is of a similar order of magnitude as the record-to-record variability. However, labelling this kind of uncertainty as epistemic or aleatory is not an objective issue, since the amount of available data in the literature does not allow quantifying the uncertainty that could be reduced with larger data bases or more refined models.
This conclusion strongly depends on the assumptions that are made on the distribution of the displacement capacity, which at present is generally defined as a drift limit, and on the type of modelling that is adopted. The uncertainty related to the displacement capacity is here determined by a statistical analysis of the cyclic quasi-static tests available in the literature. The modelling approach, disregarding the assessment of out-of-plane failure mechanisms, can overestimate the role of the in-plane displacement capacity. However, the epistemic uncertainty related to the out-of-plane behaviour, although less related to mechanical properties and harder to quantify, is considerable, for the complexity of the problem and for the simplifications that need to be assumed in the modelling.
The applied methods, in particular the PEM, when combined to a pushover analysis, are studied here as a solution with a numerical cost and a level of complexity that can be also applied in engineering practice. This seems important as the case study showed that the epistemic uncertainty related to the structural response is in the same order as the uncertainty related to the record-to-record variability..

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